Temple University Econometrics I Economics 8009

Linear Algebra Homework

1. Define A = $\begin{bmatrix} 1 & 4 & 7 \\ 3 & 2 & 5 \\ 5 & 2 & 8 \end{bmatrix}$. 1.a. Calculate the determinant of A.

1.b. Calculate the trace of A.

1.c. Find A^{-1} , the inverse of A.

2. The Cholesky decomposition of a matrix, A, is the matrix product LU = A, where L is a lower triangular matrix and U is its transpose, i.e. U = L'. Find the Cholesky

decomposition of the matrix A defined as $A = \begin{bmatrix} 25 & 7 \\ 7 & 13 \end{bmatrix}$.

3. What is the Jacobian for the following system of equations?

$$y_1 = ln\left(\frac{x_1}{x_2}\right)$$
$$y_2 = x_1 - x_2 + x_3$$
$$y_3 = x_1x_2x_3$$

4. For the matrix
$$X' = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & -2 & 3 & -5 \end{bmatrix}$$

- 4.a. Compute $P = X(X'X)^{-1}X'$.
- 4.b. Compute M = (I P)
- 4.c. Show that P is idempotent.
- 4.d. Show that M is idempotent.
- 4.e. Show that MP = 0.
- 4.f. What are the characteristic roots of P?
- 4.g. What are the characteristic roots of M?